

# Advanced Math

## 6-3

(Day 2)

## Vectors in a Plane

General unit vector - endpoint on unit circle

$$\vec{u} = (\cos \theta, \sin \theta)$$

$$\vec{i} \cos \theta + \vec{j} \sin \theta$$

Trig form of a vector -

$$\vec{v} = \|\vec{v}\| (\vec{i} \cos \theta + \vec{j} \sin \theta)$$

Write the trig form of  $\vec{v}$  at the right.

$$\|\vec{v}\| = \sqrt{2^2 + 1^2} = \sqrt{5} \quad \tan \theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.565^\circ$$

$$\vec{v} = \sqrt{5} (\vec{i} \cos 26.565^\circ + \vec{j} \sin 26.565^\circ)$$

Find a unit vector in the direction of the given vector. (pg. 541)

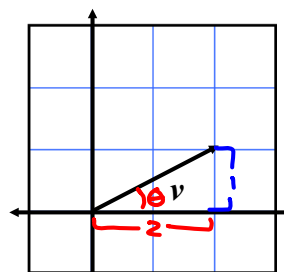
$$*) \vec{w} = \langle 3, 4 \rangle = 5 (\vec{i} \cos 53.130^\circ + \vec{j} \sin 53.130^\circ)$$

$$\|\vec{w}\| = \sqrt{3^2 + 4^2}$$

$$\tan \theta = \frac{4}{3} \quad \theta = 53.130^\circ$$

$$\text{unit vector: } (\vec{i} \cos 53.130^\circ + \vec{j} \sin 53.130^\circ)$$

Note the unit vector has magnitude 1.



Find the vector  $v$  with the given magnitude and the same direction as  $u$ .

\*)  $\|v\| = 12$   $\|u\| = \sqrt{29}$   $u = -2i - 5j = \langle -2, -5 \rangle$   $\tan \theta = \frac{-5}{-2}$  **Quad III**

$$\vec{v} = 12 (\vec{i} \cos 248.199^\circ + \vec{j} \sin 248.199^\circ)$$

$$\vec{v} = \left\langle \frac{12(-2)}{\sqrt{29}}, \frac{12(-5)}{\sqrt{29}} \right\rangle \text{ or } \left\langle \frac{-24}{\sqrt{29}}, \frac{-60}{\sqrt{29}} \right\rangle$$

$\theta = 68.199^\circ$   
**III**  $\rightarrow$  No!  $+180^\circ$   
 $248.199^\circ$

Arctan will only place answers in quadrants I and IV. You have to manually adjust the angles if they need to be in II or III by adding  $180^\circ$ .

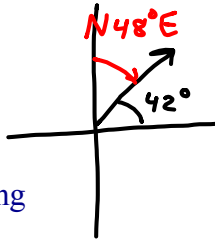
Find the magnitude and direction angle of  $v$ .

\*)  $v = 4 (\cos 42^\circ i + \sin 42^\circ j)$

$\|v\| = 4$

$\theta = 42^\circ$  or  $N48^\circ E$

Optional bearing notation.



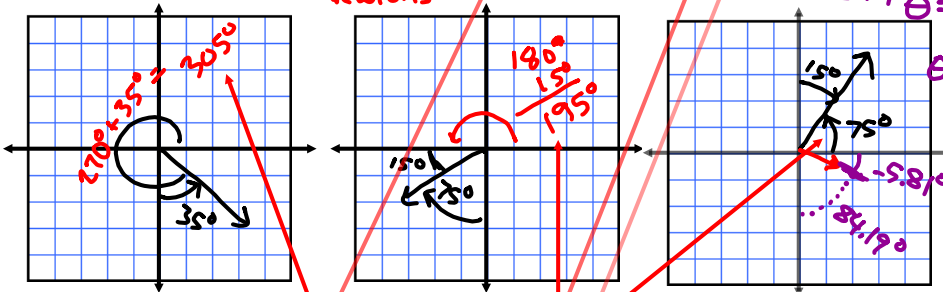
Find the resultant force acting upon an object given:

Force 1: 300N at  $S35^\circ E$   $\rightarrow 300 (\vec{i} \cos 305^\circ + \vec{j} \sin 305^\circ) = \langle 172.073, -245.746 \rangle$

Force 2: 125N at  $S75^\circ W$   $\rightarrow 125 (\vec{i} \cos 195^\circ + \vec{j} \sin 195^\circ) = \langle -120.741, -32.352 \rangle$

Force 3: 275N at  $N15^\circ E$   $275 (\vec{i} \cos 75^\circ + \vec{j} \sin 75^\circ) = \langle 71.175, 265.630 \rangle$

$(122.507)^2 + (-12.468)^2 = 123.140 \text{ N}$  @  $S84.19^\circ E$   $\leftarrow$  OR  $\langle 122.507, -12.468 \rangle$



Angles need to be measured in standard notation when using the trig form.

An object is stationary if the resultant forces acting upon it =  $\mathbf{0}$ . If a weight is suspended as shown, how much tension is in each supporting cable?

$\vec{u} + \vec{v} + \vec{w} = \vec{0}$   
 $\|\vec{u}\| (i \cos 150^\circ + j \sin 150^\circ)$   
 $\|\vec{v}\| (i \cos 45^\circ + j \sin 45^\circ)$   
 $3000 (i \cos 270^\circ + j \sin 270^\circ)$

$\textcircled{2} -\frac{\sqrt{3}}{2} \|\vec{u}\| + \frac{1}{\sqrt{2}} \|\vec{v}\| + 3000(0) = 0$

$\textcircled{3} \frac{1}{\sqrt{2}} \|\vec{v}\| = \frac{\sqrt{3}}{2} \|\vec{u}\|$

$\frac{1}{\sqrt{2}} \|\vec{v}\| = \frac{\sqrt{3}}{2} (2196.15)$

$\|\vec{v}\| = \frac{\sqrt{6}}{2} (2196) \approx 2689.73$

$\textcircled{4} \frac{1}{2} \|\vec{u}\| + \frac{1}{\sqrt{2}} \|\vec{v}\| + 3000(-1) = 0$

$\textcircled{5} \frac{1}{2} \|\vec{u}\| + \frac{\sqrt{3}}{2} \|\vec{u}\| = 3000$   
 $(\frac{\sqrt{3}+1}{2}) \|\vec{u}\| = 3000$   
 $\|\vec{u}\| = \frac{6000}{\sqrt{3}+1}$

$\textcircled{6} \approx 2196.15$

Assignment:  
 pg. 540  
 1-24 all,  
 37-42 all.